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Some Properties of the F- Structure Manifold Satisfying $F^{2k+s}+F^{s}=0$

Abstract

In this paper, we have studied various properties of the F-structure manifold satisfying $F^{2k+s}+F^{s}=0$, where K and S are positive integers and 2K≥S. The metric F- structure, f induced on each integral manifold of tangent bundle I* have also been discussed.

Keywords: Differentiable manifold, projection operators, tangent bundle and metric.

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Introduction:

Let V_n be a C^∞ differentiable manifold and F be a $C^\infty(1,1)$ tensor defined on V_n, s.t.

(1.1) $F^{2k+s}+F^{s}=0$.

We define the projection operators l and m on V_n by

(1.2) $l= -F^{2k}$, $m=I+F^{2k}$

From (1.1) and (1.2), we have

(1.3) $l+m=I$, $l^2=l$, $m^2=m$, $lm=ml=0$

$lF^s=F^sl= F^s$, $mF^s= F^sm=0$

Aim of the study

The ain of study is to develope the conditions under which recovery of (1.1) is possible.

Theorem (1.1) If rank ((F))=n, then

(1.4), $l=I$, $m=0$

Proof from the fact

(1.5) rank ((F))+ nullity ((F))= dim V_n=n

(1.6) Nullity((F))=0 =) Ker ((F)) = [0]

Thus FX=0 =) X=0

Then let $FX_1 = FX_2$

=) $F(x_1-x_2)=0$

=) X_1-X2=0 or F is 1-1.

Moreover V_n being finite dimensional, F is onto also : F is invertible =)F^k is invertible, operating F^{-k} on $F^kl=F^k$ and $mF^k=0$, we get (1.4)

Throrem (1.2) let m and F satisfy

(1.7) $m^2=m$, $mF^k = F^km=0$, $(m+F^k)(m-F^k)=I$, $mF^s=0$
then F satisfying (1.1)

Proof: $(m+F^k)(m-F^k)= I$

$m^2-mF^k+F^km-F^{2k}=I$

$m-0+0-F^{2k}=I$

$mF^s-F^{2k+s}=F^s$

$0-F^{2k+s}=F^s$

Or $F^{2k+s}+F^s=0$

Metric F Structure, If we define (2.1)
$'F(X,Y)=g(FX,Y)$ is skew symmetric then,
(2.2) $g(FX, Y)= -g(X, FY)$

Theorem (2.1) with the definitions in (2.1) and (2.2), we have

(2.3) $g(F^kX,F^ky)=(-1)^{k+1}[g(X, Y)- {}'m(X, Y)]$

Where,

(2.4) $'m(X, Y)=g(mX, Y)$

Proof, using (1.2), (1.3), (2.2) and (2.4), we have

(2.5) $g(F^kX, F^ky)=(-1)^kg(X, F^{2k}Y)$

$=(-1)^kg(X, -IY)$

$=(-1)^{k+1}(g(X, IY))$

$=(-1)^{k+1}g(X, (I-m)Y)$

$=(-1)^{k+1}[g(X,Y)-g(X,mY)]$

$=(-1)^{k+1}[g(X,Y)-g(mX,Y)]$

$=(-1)^{k+1}[g(X,Y)- {}'m(X,Y)]$

Theorem (2.2) [F, g] is not unique.

Proof (2.6), Let $\mu F'=F\mu$, $'g(X,Y)=g(\mu X,\mu Y)$

Then, from (1.1), (1.2), (1.3) and (2.6)

(2.7) $\mu F'^{2k+s}= F^{2k+s}\mu= -F^s\mu= -\mu F'^s$

Or

(2.8) $F'^{2k+s}+F'^s=0$. Also

(2.9) $'g(F'^kX, F'^kY)=g(\mu F'^kX, \mu F'^kY)$

$=g(F^k\mu X, F^k\mu Y)$

$=(-1)^kg(\mu X, F^{2k}\mu Y)$

$=(-1)^k g(\mu X, -I\mu Y)$

$=(-1)^{k+1} g(\mu X, I\mu Y)$

$=(-1)^{k+1} g(\mu X,(I-m)\mu Y)$

$=(-1)^{k+1}[g(\mu X, \mu Y)-g(\mu X, m\mu Y)]$

$=(-1)^{k+1}['g(X,Y)- {}'m'(X,Y)]$

Induced structure f, define
(3.1) $fX'=FX'$ for $x \in I^*$

Theorem (3.1) If f satisfy (3.1) and F (1.1) then $[f^k]$ is an almost complex structure.

Proof: From (1.2), (1.3) and (3.1)

(3.2) $f^{2k}IX'= F^{2k}IX'$

$= -I^2X'$

$= -IX'$.

Thus $[f^k]$ acts as an almost complex structure.

Also (3.3) $\mu I'= -\mu F'^{2k}$

$=-F^{2k}\mu$

$=I\mu$

(3.4) $\mu m' = \mu(I+F'^{2k})$

$=\mu+\mu F'^{2k}$

$=\mu+F^{2k}\mu$

$=(I+F^{2k})\mu$

$=m\mu$

Conclusion

If $m^2=m$, $mF^k=F^km=0=mF^S$, $(m+F^k)(m-F^k)=I$, then (1.1) is recovered.

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